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# Fermion mass hierarchies in low energy supergravity and superstring models<sup>1</sup>

Pierre Binétruy

*Laboratoire de Physique Théorique et Hautes Energies<sup>2</sup>  
 Université Paris-Sud, Bât. 211, F-91405 Orsay Cedex, France*

Emilian Dudas

*Service de Physique Théorique de Saclay,  
 91191 Gif-sur-Yvette Cedex, France*

## Abstract

We investigate the problem of the fermion mass hierarchy in supergravity models with flat directions of the scalar potential associated with some gauge singlet moduli fields. The low-energy Yukawa couplings are nontrivial homogeneous functions of the moduli and a geometric constraint between them plays, in a large class of models, a crucial role in generating hierarchies. Explicit examples are given for no-scale type supergravity models. The Yukawa couplings are dynamical variables at low energy, to be determined by a minimization process which amounts to fixing ratios of the moduli fields. The Minimal Supersymmetric Standard Model (MSSM) is studied and the constraints needed on the parameters in order to have a top quark much heavier than the other fermions are worked out. The bottom mass is explicitly computed and shown to be compatible with the experimental data for a large region of the parameter space.

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# 1 Introduction

One of the mysteries of the Standard Model of strong and electroweak interactions is the difference between the mass of the top quark and the mass of the other fermions. Taking as fundamental the electroweak scale  $v \simeq 250\text{GeV}$ , the top quark mass is roughly of the order  $v$ , whereas in a first approximation all the other fermions are massless. The Standard Model by itself cannot explain this puzzle ; by definition the Yukawa couplings (Yukawas) are just free parameters, and so are the physical fermion masses. Going beyond the Standard Model, Grand Unified Theories and (or) Supersymmetry give some relations between the Yukawas, but do not answer the question. The couplings are still free parameters, to be eventually determined in a more fundamental theory.

An interesting idea was recently proposed in the context of the Standard Model by Nambu [1]. Essentially the vacuum energy density is minimized with respect to the Yukawa couplings  $\lambda_i$ , all the other parameter being held fixed, including the vev's of the scalar fields. The Yukawas  $\lambda_i$  are subject to a constraint, of vanishing quadratic divergences in the Higgs sector of the theory.

This gives the so-called Veltman condition [2], which in the case of two Yukawas  $\lambda_1$  and  $\lambda_2$  gives

$$\lambda_1^2 + \lambda_2^2 = a^2, \quad (1)$$

where  $a$  is a constant. The vacuum energy to be minimized in the example chosen by Nambu is of the form

$$\mathcal{E}_0 = -A (\lambda_1^4 + \lambda_2^4) + B (\lambda_1^2 \ln \lambda_1^2 + \lambda_2^2 \ln \lambda_2^2). \quad (2)$$

Supposing for the moment that  $B = 0$  and minimizing  $\mathcal{E}_0$ , the minimum is obtained for  $(\lambda_1^2, \lambda_2^2) = (a^2, 0)$  or  $(\lambda_1^2, \lambda_2^2) = (0, a^2)$ . The configuration  $(\lambda_1^2, \lambda_2^2) = (\frac{a^2}{2}, \frac{a^2}{2})$  it is a local maximum for  $\mathcal{E}_0$  and it is the only extremum of  $\mathcal{E}_0$ .

In this approximation ( $B = 0$ ) we have a massless fermion and a massive one, with a mass fixed by the mass parameter of the Lagrangian. An important constant is the sign of  $A$ , which should be positive ; this will be an important argument in favor of supersymmetric theories later on. Adding the logarithmic terms  $B \neq 0$  has as effect producing a global minimum in  $\mathcal{E}_0$ , and the corresponding configuration is  $(\lambda_1^2, \lambda_2^2) \simeq (a^2, a^2 e^{-\frac{2a^2 A}{B}})$  and the similar one  $\lambda_1 \leftrightarrow \lambda_2$ . The ratio of the two fermion masses contains an exponential suppression factor  $e^{-\frac{a^2 A}{B}}$ , and the hierarchy is obtained if  $B \ll a^2 A$ .

The applicability of the mechanism to the standard model is under investigation [3].

In a previous paper [4] we argued that a natural framework to incorporate the Nambu idea is provided by the string effective supergravity models. In this case the spectrum of the theory includes gauge singlets  $T_i$  called moduli which describe the size and the shape of the six-dimensional compactified manifold.

Their vev's are not determined at the supergravity level and the scalar potential has flat directions due to some non compact symmetries.

The coupling constants at low-energy are functions of the moduli and consequently can be considered as dynamical variables to be determined by the low-energy dynamics, in addition to the gravitino mass  $m_{3/2}$ .

This is possible because the breaking of supersymmetry destroys in principle the non-compact symmetries and dynamically determines the moduli vev's. The minimization with respect to the gravitino mass  $m_{3/2}$  was extensively studied in the literature [5] in the content of the supergravity no-scale models [6] with one modulus field  $T$ . This case corresponds to the simplest case of Calabi-Yau type compactification [7] of the ten-dimensional heterotic string theory, but is not realistic because it only allows to obtain one generation of fermions at low energy. In more realistic models [8] we normally have several moduli and consequently more dynamical variables. The low-energy determination of the Yukawas is equivalent to the determination of the real part of the moduli fields. This hides in fact an important hypothesis, the masses of the moduli should be very small compared to the intermediate supersymmetry breaking scale. This is a nontrivial assumption and is a direct generalization of the no-scale idea developed in [5]. If the moduli masses are much greater than the electroweak scale, the moduli decouple at low-energy and the Yukawa couplings are just arbitrary, fixed parameters and not dynamical variables.

The purpose of the present paper is to present explicit examples of supergravity theories which illustrates the previous ideas. The characteristic feature of the models described below is the particular way of realizing a constraint of type (1) between Yukawas, directly at the tree level of supergravity. The constraint, independent of the mechanism of supersymmetry breaking, is valid at the Planck scale and should be run to low energy using the renormalization group (RG) equations. Another possibility, related to the moduli dependent threshold corrections in the string effective supergravities will not be discussed here. More models will be proposed with different constraints and the phenomenological consequences for the low-energy fermion spectrum will be analyzed. As a result, we will find that in most of the cases the spectrum consists of one massive fermion, all the other being massless in a certain approximation which corresponds to the case  $B = 0$  in the toy model (2) discussed by Nambu.

The article is organized as follows. In section 2 we analyze different supergravity models and the resulting constraints between the low-energy Yukawas. The simplest models proposed give multiplicative type constraints instead of the additive one in eq.(1). Models with additive constraints are somewhat more complicated and possess less symmetries.

In section 3 the different constraints will be analyzed in connexion with the minimization of the vacuum energy minimization. It is shown that for multiplicative type constraints in models with Higgs fields of the type MSSM, a condition on the dilaton field  $S$  is necessary in order to generate hierarchies.

In an example studied in section (3) this is  $(s + s^+)^2 > 4\pi^2/\ell n \frac{M_p}{\mu_0}$ , where  $\mu_0$  is the low-energy scale which sets the mass value for the massive fermion and  $M_p$  is the Planck scale.

Section 4 discusses the Minimal Supersymmetric Standard Model [9] and the phenomenological constraints necessary for the mechanism to work. It is shown that a minimal value for  $tg\beta$  of order one is sufficient to trigger the mechanism in such a way that an up-type quark be the heaviest one. Keeping only the top and bottom Yukawa couplings, we compute analytically the latter as a function of the gauge coupling at the Planck scale and of the low energy parameters. We find that it is directly proportional to the  $\mu$ -parameter of the MSSM. For  $tg\beta > 1$ , we find that the bottom mass is compatible with the experimental data for a large allowed region of the parameter space.

Finally some conclusions are drawn and prospects for future work are presented.

## 2 Dynamical Yukawa couplings and constraints in low-energy supergravity models.

In the following, we will only consider models with zero cosmological constant at the tree level. The gauge singlet fields considered in all examples will be the moduli  $T_i$  and a dilaton-like field  $S$ , common to all superstring effective supergravities. We will consider  $N = 1$  supergravity described by the Kähler function  $K$ , the superpotential  $W$  and the gauge kinetic function  $f$  [10]. We are not interested in this section in the gauge interactions and consequently we will neglect them in the analysis; we will return to this when discussing the MSSM.

Consider a string effective model containing the above-mentioned singlet fields and  $p$ -observable chiral fields  $\phi_A^i$ . The Kähler potential and the superpotential read

$$\begin{aligned} K &= K_0 + K_{A i_A} \phi^{i_A} \phi_{j_A}^+ + \dots, \\ K_0 &= -\frac{3}{n} \sum_{\alpha=1}^n \ln(T_\alpha + T_\alpha^+) - \ln(S + S^+) \\ W &= \frac{1}{3} \lambda_{i_A i_B i_C} \phi^{i_A} \phi^{i_B} \phi^{i_C}, \end{aligned} \tag{3}$$

where the dots stand for higher-order terms in the fields  $\phi^{i_A}$ . The index  $A$  in eq.(3) stands for sectors of the matter fields with different modular weights [11]. The Kähler metric depends on the moduli  $T_\alpha$ , and eventually on  $S$ . The low-energy spontaneously broken theory contains the normalized fields  $\hat{\phi}^i$  defined by  $\phi^{i_A} = (K_A^{-1/2})_{j_A}^{i_A} \hat{\phi}^{j_A}$  and the Yukawas  $\hat{\lambda}_{i_A i_B i_C}$ . In order to obtain the relation between  $\lambda_{i_A i_B i_C}$  and  $\hat{\lambda}_{i_A i_B i_C}$ , consider the scalar potential [6], which

contains the piece

$$V = e^K \left( \sum_A (K_A^{-1})_{j_A}^{i_A} D_{i_A} W \bar{D}^{j_A} \bar{W} - 3|W|^2 \right) \ni \hat{W}_{i_A} \hat{W}^{i_A} . \quad (4)$$

In eq.(4),  $D_{i_A} = \partial W / \partial \phi^{i_A} + K_{i_A} W$  and  $\hat{W} = \frac{1}{3} \hat{\lambda}_{i_A i_B i_C} \hat{\phi}^{i_A} \hat{\phi}^{i_B} \hat{\phi}^{i_C}$  is the low-energy superpotential: we restrict our attention here to the trilinear (renormalisable) couplings. Making the identifications in eq.(4) we obtain the relation

$$\hat{\lambda}_{i_A i_B i_C} = e^{i\theta_{i_A}} e^{\frac{K_0}{2}} (K_A^{-1/2})_{j_A}^{i_A} (K_B^{-1/2})_{j_B}^{i_B} (K_C^{-1/2})_{j_C}^{i_C} \lambda_{j_A j_B j_C} , \quad (5)$$

where  $\theta_{i_A}$  is an arbitrary real function of the moduli. From (5) we see that the low-energy Yukawas  $\lambda_{i_A i_B i_C}$  are functions of the moduli through the Kahler potential  $K$ . Some of the moduli are fixed to their vacuum energy values at high energies. Others may still remain undetermined at low energies and correspond to flat directions of the scalar potential. In this case, the low energy Yukawa couplings  $\hat{\lambda}_{i_A i_B i_C}$  are dynamical degrees of freedom whose precise value may be fixed by the dynamics at low energies.

Consider a model with  $M$  Yukawa couplings. As we will see in the next two sections, in order to understand dynamically the fermion mass hierarchy we are interested in models where the low energy Yukawas are not independent but subject to a certain number  $p$  of constraints. Such constraints can be written in terms of  $p$  independent functions  $F_i$ ,  $i = 1 \dots p$ , such that they read

$$F_i(\hat{\lambda}_1(T_\alpha), \dots, \hat{\lambda}_M(T_\alpha)) = C_i , \quad (6)$$

where  $C_i$  are constants which do not depend on the moduli. Differentiating (6) with respect to the moduli, we find  $p$  systems of  $n$  linear equations of  $M$  variables ( $\frac{\partial F_i}{\partial \hat{\lambda}_I}$ )

$$\sum_{I=1}^M \frac{\partial \hat{\lambda}_I}{\partial T_\alpha} \frac{\partial F_i}{\partial \hat{\lambda}_I} = 0 \quad (7)$$

which can easily be put in a matrix form. The condition to have  $p$  independent eigenvectors for this matrix equation is

$$\text{rank} \left( \frac{\partial \hat{\lambda}_I}{\partial T_\alpha} \right) = \min(M, n) - p . \quad (8)$$

Consider for the moment the case where the number of the Yukawas is equal to the number of moduli  $n = M$ . A low-energy constraint between Yukawas can be expressed mathematically in the following way. Eliminating the moduli fields  $T_i$  as functions of the Yukawas, the transformation is singular

$$\det \left( \frac{\partial \hat{\lambda}_I}{\partial T_\alpha} \right) = 0 . \quad (9)$$

The Jacobian (9) can be rewritten as follows

$$\begin{vmatrix} \sum_{\alpha} T_{\alpha} \frac{\partial \hat{\lambda}_1}{\partial T_{\alpha}} & \frac{\partial \hat{\lambda}_1}{\partial T_2} & \cdots & \frac{\partial \hat{\lambda}_1}{\partial T_n} \\ \sum_{\alpha} T_{\alpha} \frac{\partial \hat{\lambda}_M}{\partial T_{\alpha}} & \frac{\partial \hat{\lambda}_M}{\partial T_2} & \cdots & \frac{\partial \hat{\lambda}_M}{\partial T_n} \end{vmatrix} = 0 . \quad (10)$$

A natural solution for (10) is  $\sum_{\alpha} T_{\alpha} \frac{\partial \hat{\lambda}_I}{\partial T_{\alpha}} = 0$ , in which case the Yukawas  $\hat{\lambda}_{i_A i_B i_C}$  are *homogeneous functions of the moduli*. In other words, in any model with an equal number of Yukawas and moduli, if all the Yukawas are non trivial homogeneous functions, we will always have a constraint between them. This is so for many effective string models and will be our main assumption in the following.<sup>3</sup>

Define the modular weights of the matter fields as

$$T_{\alpha} \frac{\partial}{\partial T_{\alpha}} K_{A i_A}^{j_A} = n_A K_{A i_A}^{j_A} . \quad (11)$$

Using eq.(5), the homogeneity property of  $\hat{\lambda}_{i_A i_B i_C}$  translates into an equation for the original couplings  $\lambda_{i_A i_B i_C}$

$$\left( \frac{1}{2} T_{\alpha} K^{\alpha} - \frac{n_A + n_B + n_C}{2} + T_{\alpha} \frac{\partial}{\partial T_{\alpha}} \right) \lambda_{i_A i_B i_C} = 0 . \quad (12)$$

For theories with zero cosmological constant at tree level  $T_{\alpha} K^{\alpha} = -3$ . Denoting by  $N_{ABC}$  the modular weight of the string couplings  $\lambda_{i_A i_B i_C}$ , eq.(12) reduces to

$$n_A + n_B + n_C = -3 + 2N_{ABC} . \quad (13)$$

This equation is sufficient to guarantee the existence of constraints between Yukawa couplings. Since it depends only on the modular weights of the fields and their couplings, it proves to be useful to construct explicit models.

A simple case of interest is  $N_{ABC} = 0$  and  $K_{A i_A}^{j_A} = c_A t_A^{n_A} \delta_{i_A}^{j_A}$  where  $c_A$  are constants and  $t_A = T_A + T_A^+$ . The homogeneity property becomes explicit and the equation (5) becomes

$$\hat{\lambda}_{i_A i_B i_C} = (s c_A c_B c_C)^{-\frac{1}{2}} \prod_{\alpha} \left( \frac{t_{\alpha}}{t_C} \right)^{-\frac{3}{2n}} \left( \frac{t_A}{t_C} \right)^{-\frac{n_A}{2}} \left( \frac{t_B}{t_C} \right)^{-\frac{n_B}{2}} \lambda_{i_A i_B i_C} , \quad (14)$$

where  $s = S + S^+$ . A very particular case is  $N_{ABC} = 0$  and  $n_A = n_B = n_C = -1$ , in which case we include the matter fields  $\phi^{i_A}$  in the no-scale structure of the

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<sup>3</sup>Similarly, the condition to have  $p$  constraints in the general case  $n \neq M$  is easy to find. We must consider all the quadratic matrices  $[min(M, n)] \times [min(M, n)]$  constructed from the matrix  $[\partial \hat{\lambda}_I / \partial T_{\alpha}]$  and impose the condition that the rank of all of them be  $min(M, n) - p$ . As above, this is so if any subset of  $min(M, n) - p + 1$  Yukawa couplings obey homogeneity properties with respect to any subset of  $min(M, n) - p + 1$  moduli.

moduli. The simple models analyzed below will have this property which is typical of many compactifications of the ten-dimensional heterotic string theory.

We can easily compare the number of degrees of freedom at low energy ( $M + 1$ ) (one degree of freedom is the gravitino mass) and high-energy ( $n$ ) (we consider only the real part of the moduli; the imaginary part will play no role in the determination of the Yukawas in our examples). In order to completely fix the moduli vev's we must satisfy the inequality  $M + 1 \geq n$ . For  $M = n$  the vev's are fixed and moreover we have one constraint between the Yukawas. This is the most interesting situation which, as emphasized above, will be our main concern.

The symmetries of the supergravity models will be essential in order to restrict the class of possible constraints. An important invariance is provided by the Kähler transformations

$$\begin{aligned} K(z, z^+) &\rightarrow K(z, z^+) + F(z) + F^+(z^+) , \\ W(z) &\rightarrow e^{-F(z)} W(z) , \end{aligned} \tag{15}$$

where  $F(z)$  is an analytic function of the set of chiral superfields  $z$ . If we restrict our attention to moduli dependent functions, this transformation acts on the low energy Yukawas as a  $U(1)$  transformation :

$$\hat{\lambda}_{i_A i_B i_C} \rightarrow e^{-i \text{Im} F} \hat{\lambda}_{i_A i_B i_C} . \tag{16}$$

The transformation (15) allows us to eliminate the phase  $\theta_{i_A}$  in eq.(5) and tells us that the constraint must always contain the combination  $\hat{\lambda} \hat{\lambda}^+$ . This is not very restrictive but it ensures that our final results are Kähler invariant.

A more powerful constraint is obtained if we impose the target-space duality symmetries  $SL(2, Z)$

$$T_\alpha \rightarrow \frac{a_\alpha T_\alpha - i b_\alpha}{i c_\alpha T_\alpha + d_\alpha} , \quad a_\alpha d_\alpha - b_\alpha c_\alpha = 1, \quad a_\alpha, \dots, d_\alpha \in Z , \tag{17}$$

for every moduli  $T_\alpha$ . Since

$$T_\alpha + T_\alpha^+ \rightarrow \frac{T_\alpha + T_\alpha^+}{|i c_\alpha T_\alpha + d_\alpha|^2} , \tag{18}$$

it can be viewed as a particular type of Kähler transformations, acting explicitly on the fields  $\phi^{i_A}$ . In effective string theories of the orbifold type [11], the observable fields  $\phi^{i_A}$  and the Kähler metric  $K_{A i_A}^{j_A}$  transform under (17) as

$$\begin{cases} \phi^{i_A} &\rightarrow \phi^{i_A} / (i c_{\alpha_A} T_{\alpha_A} + d_{\alpha_A})^{n_A} \\ K_{A i_A}^{j_A} &\rightarrow |i c_{\alpha_A} T_{\alpha_A} + d_{\alpha_A}|^{2n_A} K_{A i_A}^{j_A} , \end{cases} \tag{19}$$

where  $T_{\alpha_A}$  is the moduli containing  $\phi^{i_A}$  in its no-scale structure (explicit examples will be given below). Hence the low-energy fields  $\hat{\phi}^{i_A} = (K_A^{1/2})_{i_B}^{i_A} \phi^{i_B}$  are duality invariant. For the model defined by eq.(3), the Kähler potential  $K$  transforms as follows

$$K \rightarrow K + \frac{3}{n} \sum_{\alpha_A=1}^n \ln |ic_{\alpha_A} T_{\alpha_A} + d_{\alpha_A}|^2. \quad (20)$$

Defining the function  $F_{\alpha} \equiv \frac{3}{n} \ln(ic_{\alpha} T_{\alpha} + d_{\alpha})$ , the Yukawa transformation law is obtained from eq. (5). Assuming that the original Yukawa couplings  $\lambda_{i_A i_B i_C}$  are modular invariant ( $N_{ABC} = 0$ ), one finds for the low energy couplings

$$\hat{\lambda}_{i_A i_B i_C} \rightarrow \left( \prod_{\alpha_A=1}^n e^{\frac{F_{\alpha_A} + F_{\alpha_A}^+}{2}} \right) e^{-\frac{n}{6}(F_{\alpha_A} n_A + F_{\alpha_B} n_B + F_{\alpha_C} n_C + h.c.)} \hat{\lambda}_{i_A i_B i_C}. \quad (21)$$

We can easily see that a theory which is completely duality invariant is too restrictive for us. The correct transformation of the superpotential  $W(z)$  in eq.(15) gives the equality

$$\sum_{\alpha_A=1}^n F_{\alpha_A} = \frac{n}{3} (F_{\alpha_A} n_A + F_{\alpha_B} n_B + F_{\alpha_C} n_C). \quad (22)$$

This tells us that in this case the Yukawa couplings  $\hat{\lambda}_{i_A i_B i_C}$  are duality invariant, *i.e.* invariant under (21) and at the tree level of supergravity with no threshold corrections this means that they do not depend on the moduli at all.

The most important ingredient in the construction of realistic supergravity theories is the geometrical structure of the Kahler potential, transforming like (15). The scalar fields span homogeneous spaces of the coset type, as in the examples presented below. We will consider superpotentials which present only a part of the symmetries of the Kahler function  $K$ , which are sufficient to guarantee the existence of the flat directions and a zero cosmological constant. This happens for example in the four-dimensional  $N = 1$  string constructions with spontaneous supersymmetry breaking at the tree level [12]. This mechanism can be formulated in the orbifold string constructions and the superpotential modification associated with supersymmetry breaking violates the target space duality [13].

We will use in this sense the notion of duality invariant models in the rest of this paper. Note from eq.(5) that the superpotential does not appear explicitly in the ratio  $\hat{\lambda}_{i_A i_B i_C} / \lambda_{i_A i_B i_C}$ , giving a simple geometric interpretation for this ratio.

An allowed constraint involves a duality invariant combination of  $\hat{\lambda}_{i_A i_B i_C}$ . A simple inspection of eq.(21) is sufficient to convince ourselves that only

multiplicative-type constraints can be duality invariant, due to the exponential transformation law. For example, if  $\hat{\lambda}_{i_A i_B i_C} \neq 0$  for all  $i_A, i_B, i_C$  we get in an obvious way the constraint

$$\prod_{i_A i_B i_C} \hat{\lambda}_{i_A i_B i_C} = cst. \quad (23)$$

In all the models discussed below, however, we have some  $\hat{\lambda}_{i_A i_B i_C} = 0$  and eq.(23) does not apply directly.

The simplest example contains two moduli  $T_1, T_2$ , the dilaton  $S$  and two observable fields  $\phi^i$ . The model is defined by

$$\begin{aligned} K &= -\frac{3}{2} \ln(t_1 - |\phi_1|^2) - \frac{3}{2} \ln(t_2 - |\phi_2|^2) - \ln s, \\ W &= \frac{1}{3} \lambda_1 \phi_1^3 + \frac{1}{3} \lambda_2 \phi_2^3 + W(S), \end{aligned} \quad (24)$$

where  $t_i = T_i + T_i^+$ ,  $s = S + S^+$  and  $W(S)$  is a non-perturbative contribution to  $W$  which fixes the value of  $S$  and simultaneously breaks supersymmetry, as in the usual gaugino condensation scenario [14].

The tree-level scalar potential is given by the expression

$$V_0 = \frac{1}{s(t_1 - |\phi_1|^2)^{\frac{3}{2}}(t_2 - |\phi_2|^2)^{\frac{3}{2}}} \left\{ |S \partial W / \partial S - W|^2 + \frac{2}{3} \sum_{i=1,2} (t_i - |\phi_i|^2) |\partial W / \partial \phi_i|^2 \right\} \quad (25)$$

The minimum is reached for  $\phi^i = 0$ ,  $S \partial W / \partial S - W = 0$  and  $T_i$  undetermined, with a zero cosmological constant.  $V_0$  has flat directions in the  $(T_1, T_2)$  plane and the Kähler function parametrizes a  $[SU(1,2)/U(1) \times SU(2)]^2 \times SU(1,1)/U(1)$  Kähler manifold. The superpotential  $W$  breaks the  $[SU(1,2)]^2$  symmetry associated with the moduli down to  $U(1)^2 \times$  diagonal dilatation. The residual symmetry is written explicitly in eq.(42) and is spontaneously broken together with supersymmetry. In the low energy limit  $M_P \rightarrow \infty$ , the supersymmetric scalar potential reads

$$V_0 = \hat{\lambda}_1^2 |\hat{\phi}_1|^4 + \hat{\lambda}_2^2 |\hat{\phi}_2|^4. \quad (26)$$

The low energy Yukawas as functions of the high-energy  $\lambda_i$  read from eq.(5)

$$\begin{aligned} \hat{\lambda}_1^2 &= \frac{8}{27} \frac{1}{s} \left( \frac{t_1}{t_2} \right)^{3/2} \lambda_1^2, \\ \hat{\lambda}_2^2 &= \frac{8}{27} \frac{1}{s} \left( \frac{t_2}{t_1} \right)^{3/2} \lambda_2^2. \end{aligned} \quad (27)$$

They are homogeneous functions of the moduli and, consequently, the Jacobian  $\det \left( \frac{\partial \hat{\lambda}_i}{\partial t_\alpha} \right) = 0$  as can be explicitly verified in eq.(27).  $\hat{\lambda}_i$  are dynamical variables

at low energy, together with the gravitino mass  $m_{3/2}^2 = |W|^2/(st_1^{3/2}t_2^{3/2})$ . The constraint between Yukawas is obvious from eq.(27)

$$\hat{\lambda}_1 \hat{\lambda}_2 = \frac{8}{27} \frac{1}{s} \lambda_1 \lambda_2 \equiv \tilde{a}^2 = \text{fixed} . \quad (28)$$

This eliminates one of two Yukawas as a dynamical variable and leaves us with two variables, corresponding to the two original moduli  $T_1, T_2$ . The minimization process at low energy will partially fix the vacuum state and lift the flat directions corresponding to  $\text{Re } T_1$  and  $\text{Re } T_2$ . We are still left with flat directions for the imaginary parts of the moduli,  $\text{Im } T_1$  and  $\text{Im } T_2$ .

Equation (28) is valid at the Planck scale  $M_p$ . In the effective theory at lower scales  $\mu$  we must use the renormalization group (RG) equations in order to express it as a function of  $\hat{\lambda}_i(\mu)$ . This analysis, the comparison with a Veltman-type constraint and the phenomenological consequences of minimization will be analyzed in the next section.

The importance of putting  $\phi_1$  and  $\phi_2$  in different no-scale structures can be seen by considering a slight modification of eq.(24), with the same superpotential  $W$  and the Kähler potential

$$K = -\frac{3}{2} \ln(t_1 - |\phi_1|^2 - |\phi_2|^2) - \frac{3}{2} \ln(t_2) - \ln s . \quad (29)$$

In this case the low energy couplings have the same dependence on the moduli

$$\begin{aligned} \hat{\lambda}_1^2 &= \frac{8}{27} \frac{1}{s} \left( \frac{t_1}{t_2} \right)^{3/2} \lambda_1^2 , \\ \hat{\lambda}_2^2 &= \frac{8}{27} \frac{1}{s} \left( \frac{t_1}{t_2} \right)^{3/2} \lambda_2^2 \end{aligned} \quad (30)$$

and the analog of the constraint (28) is now a proportionality relation

$$\frac{\hat{\lambda}_1^2}{\lambda_1^2} = \frac{\hat{\lambda}_2^2}{\lambda_2^2} . \quad (31)$$

As we will see in the next section, this kind of proportionality does not lead to any hierarchy of couplings. In what follows, we will consequently put in *different* no-scale structures the different quark-type fields among which we want to generate a hierarchy.

The generalization to more couplings of the model in eq.(24) is straightforward. The theory is described by

$$\begin{aligned} K &= -\frac{3}{n} \sum_{i=1}^n \ln(t_i - |\phi_i|^2) - \ln s , \\ W &= \frac{1}{3} \sum_i \lambda_i \phi_i^3 . \end{aligned} \quad (32)$$

The low-energy Yukawas are given by (from now on we will take  $\hat{\lambda}$  real)

$$\hat{\lambda}_i^2 = \left(\frac{n}{3}\right)^3 \frac{t_i^3}{s \prod_{j=1}^n t_j^{\frac{3}{n}}} \lambda_i^2 \quad (33)$$

and the resulting constraint is

$$\hat{\lambda}_1 \cdots \hat{\lambda}_n = \left(\frac{n}{3}\right)^{\frac{3n}{2}} \frac{\lambda_1 \cdots \lambda_n}{s^n} = \text{fixed} . \quad (34)$$

The important point to retain is that different observable fields  $\phi^i$  are included in different no-scale structures, corresponding to different moduli. Including two observable fields in the same moduli structure will produce proportional Yukawas and not multiplicative constraints between them. In the following section we will see that multiplicative constraints are essential in this framework to understand why one low-energy fermion is much heavier than the other fermions.

More possibilities are left when one introduces scalar fields to play the role of the Higgs fields of the MSSM. For example, a model defined by

$$\begin{aligned} K &= -\frac{3}{n} \sum_{i=1}^n \ln(t_i - |\phi_i|^2 - |H_i|^2) - \ln s , \\ W &= \frac{1}{2} \sum \lambda_i \phi_i^2 H_i , \end{aligned} \quad (35)$$

with an observable field  $\phi^i$  and a Higgs  $H_i$  corresponding to one moduli  $T_i$  gives the same constraint as the preceding model, eq.(34).

If we want to couple more observable fields to the same Higgs field, we can consider the model

$$\begin{aligned} K &= -\frac{3}{n} \sum_{i=1}^{n-1} \ln(t_i - |\phi_i|^2) - \frac{3}{n} \ln(t_n - |H|^2) - \ln s , \\ W &= \frac{1}{2} (\lambda_1 \phi_1^2 + \cdots + \lambda_{n-1} \phi_{n-1}^2) H + \frac{1}{3} \lambda_n H^3 . \end{aligned} \quad (36)$$

We introduced a special modulus for the Higgs  $H$  in order not to break the permutation symmetry between the observable fields and to keep at the same time the duality symmetries. The constraint is easily obtained by defining ratios of moduli of the type  $A_i = t_i/t_1, i > 1$ . In this way the low-energy Yukawas depend only on  $(n-1)$  variables ,

$$\begin{cases} \hat{\lambda}_1^2 = \left(\frac{n}{3}\right)^3 \frac{1}{s} \frac{A_n}{(A_2 \cdots A_n)^{\frac{3}{n}}} \lambda_1^2 \\ \hat{\lambda}_i^2 = \left(\frac{n}{3}\right)^3 \frac{1}{s} \frac{A_i^2 A_n}{(A_2 \cdots A_n)^{\frac{3}{n}}} \lambda_i^2 , \quad i = 2, \dots, n . \end{cases} \quad (37)$$

Eliminating  $A_i = (\lambda_1/\lambda_i)(\hat{\lambda}_i/\hat{\lambda}_1)$ , we get the constraint

$$\hat{\lambda}_1 \cdots \hat{\lambda}_{n-1} = C \hat{\lambda}_n^{\frac{n}{3}-1} . \quad (38)$$

where  $C = \left(\frac{n}{3}\right)^n \left(\frac{1}{s}\right)^{n/3} \lambda_1 \cdots \lambda_{n-1} \lambda_n^{1-n/3}$ . This constraint is multiplicative and symmetric in the Yukawas  $\hat{\lambda}_1 \cdots \hat{\lambda}_{n-1}$  (asymmetric constraints are easy to obtain from asymmetries in the Kähler potential) and only  $\hat{\lambda}_n$  plays a particular role. The multiplicative constraints satisfy automatically the Kähler invariance, eq.(15). In all our examples this will be automatic, because the models are Kähler invariant (in the specific sense described before) at tree level.

We can consider a model which is the closest one to the minimal non-minimal extension of MSSM [18], described by

$$\left\{ \begin{array}{l} W = \frac{1}{2} \sum_i (\lambda_i \phi_i^2) H_1 + \frac{1}{2} \sum_\alpha (\lambda_\alpha \phi_\alpha^2) H_2 + \lambda_Y H_1 H_2 Y + \frac{k}{3} Y^3 , \\ K = -\frac{3}{n+2} \sum_{i=1}^{n_1} \ln(t_i - |\phi_i|^2) - \frac{3}{n+2} \sum_{\alpha=1}^{n_2} \ln(t_\alpha - |\phi_\alpha|^2) \\ \quad - \frac{3}{n+2} \ln(t_{n+1} - |H_1|^2 - |H_2|^2) - \frac{3}{n+2} \ln(t_{n+2} - |Y|^2) , \end{array} \right. \quad (39)$$

with  $M_1 + M_2 = n$ . As all the previous examples, this model has a duality invariance with respect to all moduli  $T_i$  and the number of moduli is equal to the number of Yukawas. The two Higgs are put in the same moduli structure  $T_{n+1}$ , but as before any observable quark field  $\phi^i$  is associated with a different modulus. The constraint is computed in the same way as in the model of eqs.(36, 38). The result is

$$\hat{\lambda}_1 \cdots \hat{\lambda}_n = \text{fixed} \times \frac{\hat{\lambda}_Y^{\frac{n-2}{2}}}{\hat{k}^{\frac{n+2}{6}}} \quad (40)$$

and is symmetric in  $\hat{\lambda}_1, \dots, \hat{\lambda}_n$ .

All the examples discussed above are simple and have the duality invariances, eq.(17). The symmetry group is non compact  $[SL(2, \mathbf{R})]^n$ , where  $n$  is the number of moduli, and in particular this gives flat directions in the scalar potential. They can, due to their symmetry, eventually be considered as the point-field limit of compactified superstring models.

In fact, it is possible to construct models such that the constraint is exactly in the Veltman form, if we abandon the duality symmetries. A general expression for a Veltman type condition can be easily obtained for the generic class of models defined in eq.(3) using eq.(5). The result is

$$\hat{\lambda}_{i_A i_B i_C} \hat{\lambda}^{i_A i_B i_C +} = e^{K_0} (K_A^{-1})_{j_A}^{i_A} (K_B^{-1})_{j_B}^{i_B} (K_C^{-1})_{j_C}^{i_C} \lambda_{i_A i_B i_C} \lambda^{j_A j_B j_C +} . \quad (41)$$

Using eq.(18-20) we can explicitly check that such a constraint violates the duality symmetries, as proved more generally in eq.(21 - 23).

It is nonetheless easy to construct models with the remnant symmetry  $U(1)^n \times$  diagonal scale symmetry in the Kähler potential. The transformations of the moduli are

$$\left\{ \begin{array}{l} T_\alpha \rightarrow T_\alpha + i b_\alpha \\ T_\alpha \rightarrow a T_\alpha \end{array} \right. \quad (42)$$

They are sufficient in order to get the flat directions and to forbid renormalizable terms in the superpotential  $W$ . The global scale invariance is characteristic of string models in the limit of large moduli.

A first example has no Higgs field and the superpotential is given in eq.(32). The required Kähler potential is

$$K = -\frac{3}{n} \sum_i \ell n \left[ t_i - \left( \frac{t_i^2 \sum_{j=1}^n t_j}{\prod_{k=1}^n t_k^{\frac{3}{n}}} \right)^{\frac{1}{3}} |\phi_i|^2 \right] - \ell n(S + S^+) . \quad (43)$$

Putting  $\lambda_i = \lambda$  for simplicity, the constraint is

$$\sum_{i=1}^n \hat{\lambda}_i^2 = \frac{\lambda^2}{s + s^+} \quad (44)$$

and it is of the Veltman type, eq.(1).

A second example contains a Higgs field  $H$  plus  $n$  quark fields  $\phi_i$ . The model is defined by

$$\begin{aligned} K &= -\frac{3}{n} \sum_{i=1}^n \ell n \left( t_i - |\phi_i|^2 - (t_i^3 / \prod_{j=1}^n t_j^{\frac{3}{n}}) |H|^2 \right) - \ell n(S + S^+) , \\ W &= \frac{1}{2} (\sum_i \lambda_i \phi_i^2) H . \end{aligned} \quad (45)$$

The resulting constraint is the same as in eq.(44). We consider that the last two models are less interesting in that they have less symmetries and cannot be considered as effective orbifold models with spontaneous supersymmetry breaking, as in the previous examples. In the following section we will concentrate on the multiplicative constraints.

Some remarks should be made about the induced soft supersymmetry breaking terms. In the so-called large hierarchy compatible supergravities [19] all of them depend on the gravitino mass and on the scaling weight of the metric for the chiral fields. They are independent of the Yukawas and should be treated as dynamical variables if one minimizes with respect to  $m_{3/2}$ . In the gaugino condensation scenario, at tree level none of these terms appear in the observable sector. At the one loop level, gaugino masses are induced [15] and they depend on the Planck and gravitino mass, but not on Yukawas. Through gauge interactions, they produce in principle all the other breaking terms, which in the lowest order will not depend on Yukawas. In the next paragraphs we will always consider the soft terms as being independent of the Yukawas.

In order to simplify as much as possible the analysis, we suppose that only the Yukawas of the quark type fields are dynamical and the others are fixed. This is equivalent to fixing some moduli fields which do not contain the quark fields in their no-scale structure. For example, in the model defined in eq.(36), fixing the moduli  $T_n$  which contains the Higgs field  $H$  in the no-scale structure is equivalent to fixing the coupling  $\lambda_n$ . In the following we will suppose, for simplicity reasons,

that the moduli related to the Higgs type fields and, consequently, the Higgs Yukawa self-interactions, are fixed. The essential difference between the quark type Yukawas and the Higgs self-interactions is that the second ones appear in the vacuum energy already at tree level, for multiplicative type constraints. The first ones contribute only at one loop and the generated hierarchy is radiatively induced, if they are considered as dynamical variables.

### 3 Constraints between Yukawa couplings and fermion mass hierarchies.

We start by clarifying the minimization with respect to the Yukawas, in connection with the moduli determination of the effective superstring theory.

Clearly, for our procedure to make sense, we must assume that some moduli fields remain undetermined down to low energies, *i.e.* down to energies below the supersymmetry breaking scale. Then the orientation of the vacuum in the corresponding flat directions of the potential will take place according to the details of the low energy theory. This, rephrased in the language of the low energy Yukawa couplings, will lead to the dynamical determination of some of these Yukawa couplings. Denoting by  $V_0(\hat{\lambda}_i, \phi, m_{3/2})$  the scalar potential in the observable sector, we have to minimize it with respect to the yet undetermined moduli fields:

$$\sum_{I=1}^M \frac{\partial \hat{\lambda}_I}{\partial T_\alpha} \frac{\partial V_0}{\partial \hat{\lambda}_I} = 0. \quad (46)$$

In the presence of the constraints, the matrix  $\frac{\partial \hat{\lambda}_I}{\partial T_\alpha}$  is degenerate, as expressed in eq.(8) so the minimization with respect to the Yukawas is subject to constraints as well. Obviously, the minimization should be performed taking into account the constraints (6).

Using the RG invariance of  $V_0$ , we can write eq.(46) as a function of the low energy Yukawas  $\hat{\lambda}_I(\mu_0)$

$$\frac{\partial \hat{\lambda}_I(M_P)}{\partial T_\alpha} \frac{\partial \hat{\lambda}_J(\mu_0)}{\partial \hat{\lambda}_I(M_P)} \frac{\partial V_0}{\partial \hat{\lambda}_J(\mu_0)} = 0. \quad (47)$$

This equation has two solutions :

i)  $\frac{\partial \hat{\lambda}_J(\mu_0)}{\partial \hat{\lambda}_I(M_P)} = 0$ , for any  $I, J$ . This may happen if  $\hat{\lambda}_I(M_P) \rightarrow \infty$ , in which case all the Yukawas reach their maximally allowed values at  $\mu_0$ . This is the approach followed in [22], for example.

ii)  $\frac{\partial \hat{\lambda}_J(\mu_0)}{\partial \hat{\lambda}_I(M_P)} \neq 0$ ,  $rank(\frac{\partial \hat{\lambda}_I(M_P)}{\partial T_\alpha} \frac{\partial \hat{\lambda}_J(\mu_0)}{\partial \hat{\lambda}_I(M_P)}) = M - p$ .

In this case, minimizing with respect to the moduli is equivalent to minimizing  $V_0$  with respect to the Yukawas at  $\mu_0$ , using as constraints

$$F_i \left( \hat{\lambda}(M_P) [\hat{\lambda}(\mu_0)] \right) = C_i . \quad (48)$$

This solution is more interesting for generating the mass hierarchy between the fermions and corresponds generally to the Nambu mechanism discussed in the Introduction. The constraints  $F_i$  are generically more complicated than the simple eq.(1), but the qualitative results are similar.

We will work under the hypothesis that the second solution ii) corresponds to the real vacuum and that a non-trivial minimization must be performed at low energy  $\mu_0$ .

An additive constraint of the Veltman type (1) was analyzed in [1] and it was shown to produce a hierarchy between the fermion masses, irrespective of the value of the constant  $a$ .

We now analyze in detail the multiplicative constraints of the type (34). If we are interested in the effective spontaneously broken supersymmetric theory at a scale  $\mu_0$ , we must run eq.(34) from  $M_P$  to  $\mu_0$  using the renormalization group (RG) equations for the effective renormalizable theory.

To compute the vacuum energy at the low-energy scale  $\mu_0 \sim M_{susy}$  we proceed in the usual way. Using boundary values for the independent model parameters at the Planck scale  $M_P$  (identified here with the unification scale), we evolve the running parameters down to the scale  $\mu_0$  using the RG equations and use the effective potential approach [16]. The one-loop effective potential has two pieces

$$V_1(\mu_0) = V_0(\mu_0) + \Delta V_1(\mu_0) , \quad (49)$$

where  $V_0(\mu_0)$  is the renormalization group improved tree-level potential and  $\Delta V_1(\mu_0)$  summarizes the quantum corrections given by the formula

$$\Delta V_1(\mu_0) = (1/64\pi^2) \text{Str}M^4 \left( \ln \frac{M^2}{\mu_0^2} - \frac{3}{2} \right) . \quad (50)$$

In (50)  $M$  is the field-dependent mass matrix and all the parameters are computed at the scale  $\mu_0$ . The vacuum state is determined by the equation  $\partial V_1 / \partial \phi_i = 0$ , where  $\phi_i$  denotes collectively all the fields of the theory. The vacuum energy is simply the value of the effective potential computed at the minimum. Let us define an 'average' mass  $\bar{m}$ . <sup>4</sup> In the following the minimization process is always performed for  $\mu_0^2 > \bar{m}^2 e^{-3/2}$ .

For the toy model with no Higgs, eq.(24), the RG equations for  $\lambda_1$  and  $\lambda_2$  are completely decoupled and can be easily integrated. The constraint (28) can

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<sup>4</sup> Writing the decomposition  $\text{Str}M^4 \left( \ln M^2 / \mu_0^2 - 3/2 \right) = \text{Str}M^4 \left( \ln \bar{m}^2 / \mu_0^2 - 3/2 \right) + \text{Str}M^4 \ln M^2 / \bar{m}^2$ , one can choose  $\bar{m}$  such that the second term in the decomposition, containing logarithmic terms in the Yukawas, has a minimal contribution.

then be rewritten in the form (two couplings)

$$\frac{\lambda_1^2(\mu_0)}{1 - \frac{3\lambda_1^2(\mu_0)}{16\pi^2} \ln \frac{M_P}{\mu_0}} \quad \frac{\lambda_2^2(\mu_0)}{1 - \frac{3\lambda_2^2(\mu_0)}{16\pi^2} \ln \frac{M_P}{\mu_0}} = \tilde{a}^4, \quad (51)$$

where  $\lambda_i(\mu_0)$  are the effective couplings at the scale  $\mu_0$  and we omitted the hat notation for simplicity. If we were in a perturbative regime  $\frac{3\lambda_i^2(\mu_0)}{16\pi^2} \ln \frac{M_P}{\mu_0} \ll 1$ , an expansion of eq.(51) would give us a Veltman-type constraint (1) with  $a^2 = \frac{16\pi^2}{3 \ln \frac{M_P}{\mu_0}}$ .

In order to have the Nambu mechanism, we will suppose that at a low-energy  $\mu_0$  the fields  $\phi_i$  decouple and can be fixed at their minimum, their masses being larger than that of the moduli (for realistic models such as the MSSM, or more generally models with Higgs fields, this hypothesis is not necessary). The vacuum energy at a low energy scale  $\mu_0$  for a theory with soft scalar masses  $\mu$  is given by [4]

$$\mathcal{E}_0(\mu_0) = -M^4 [\xi_1(1 - x_1) + \xi_2(1 - x_2)](\mu_0), \quad (52)$$

where we neglected the logarithmic terms in the Yukawas and we used the notations

$$x_i = \frac{\lambda_i^2}{8\pi^2} \ln \tilde{\mu}_0^2, \quad \xi_i = \frac{1}{x_i} \left( \frac{1 - 2x_i}{1 - x_i} \right)^2, \quad i = 1, 2 \quad (53)$$

and  $M^4 = \frac{\mu^4}{32\pi^2} \ln \tilde{\mu}_0^2$ , with  $\tilde{\mu}_0^2 = \mu_0^2 e^{3/2} / \bar{m}^2$ .

The lowest-energy configuration for (51) and (52) is obtained for  $\xi_1 \rightarrow \infty$  ( $x_1 \rightarrow 0$ ) and  $\lambda_2^2(\mu_0) = \frac{16\pi^2}{3 \log \frac{M_P}{\mu_0}}$  or the solution obtained by exchanging  $\lambda_1 \leftrightarrow \lambda_2$ .

In this approximation one fermion remains massless whereas the other one becomes massive. Its corresponding Yukawa coupling reaches the “triviality bound” [17], characterized by  $\lambda_2^2(M_P) \gg 1$ . We are thus not in a perturbative regime for  $\lambda_2$  and we cannot develop in a series eq.(51). Even in the nonlinear form, however, eq.(51) produces the same mechanism of generating hierarchies. The difference with respect to the Veltman-type constraint (1) is that the mass of the massive fermion is now controlled by the triviality bound. Taking  $n$  Yukawa couplings constrained by eq.(34) will always give  $n - 1$  massless fermions and a massive one.

One can relate the determination of the couplings (or, equivalently, the moduli) to the spontaneous breakdown of the residual diagonal dilatation symmetry (42), which acts on the fields as  $\phi_i \rightarrow a^{1/2} \phi_i$ . Soft mass terms in the tree-level potential induce nonzero vev’s  $\langle \phi_i \rangle$  which break the dilatation symmetry. Supersymmetry breaking is, consequently, an important ingredient in the mechanism. Remark that in the absence of the constraint (51), the minimization process would force both couplings  $x_1, x_2$  to vanish thus forbidding any hierarchy between the fermion masses.

We now turn to a more realistic case containing Higgs fields and described by eq.(36) with  $n = 3$  and the multiplicative constraint (38). It is for example the case of the MSSM when one considers only two u-type quarks and it is the ideal example for understanding the hierarchies between the fermion generations. In order to have complete analytical expressions, we will limit ourselves to the case  $\lambda_3 = 0$ , which reproduces the essential features of the mechanism.

Using the effective potential formalism and neglecting the logarithmic corrections we can write the vacuum energy (the dependence in  $\lambda_1, \lambda_2$ ) as

$$\mathcal{E}_0 = -A(\lambda_1^2 + \lambda_2^2) \quad (54)$$

with  $A > 0$ . To obtain this, we add the most general soft breaking terms at low-energy  $\mu_0$  and compute  $\text{Str } M^4$ , where

$$\text{Str} M^n = \sum_J (-1)^{2J} (2J+1) \text{Tr} M_J^n . \quad (55)$$

The quark-type fields  $\phi_i$  have zero vev's and the Higgs vev's  $\langle H_i \rangle$  are held fixed and independent of  $\lambda_i$ 's. The expression (54) is typical of the MSSM, which will be studied in detail in the next paragraph. The first step is to run the constraint (38) for  $n = 3$  from  $M_P$  to  $\mu_0$  using the RG equations

$$\begin{cases} \mu \frac{d}{d\mu} \lambda_1 = \frac{\lambda_1}{32\pi^2} (5\lambda_1^2 + \lambda_2^2) \\ \mu \frac{d}{d\mu} \lambda_2 = \frac{\lambda_2}{32\pi^2} (5\lambda_2^2 + \lambda_1^2) . \end{cases} \quad (56)$$

For arbitrary initial values  $\lambda_i(M_P)$  that fulfill eq.(38) this cannot be done analytically. Exact integration of (56) is possible if  $\lambda_1(M_P) = \lambda_2(M_P)$  and an approximate solution is found if  $\lambda_2(M_P) \ll \lambda_1(M_P)$ . Obviously, the corresponding Yukawas at low energy  $\mu_0$  will be equal  $\lambda_1(\mu_0) = \lambda_2(\mu_0)$  in the first case and will satisfy  $\lambda_2(\mu_0) \ll \lambda_1(\mu_0)$  in the second. We are then able to compare the vacuum energy for the two configurations and to decide whether the minimum corresponds to a symmetric or an asymmetric solution. A complete numerical analysis will follow for the whole set of boundary conditions  $\lambda_i(M_P)$ .

i)  $\lambda_1(M_P) = \lambda_2(M_P)$ . Due to the symmetry of eq.(56) we will have  $\lambda_1(\mu_0) = \lambda_2(\mu_0) \equiv \lambda(\mu_0)$ . A straightforward integration of eq.(56) gives

$$\lambda^2(M_P) = \frac{\lambda^2(\mu_0)}{1 - \frac{3\lambda^2(\mu_0)}{8\pi^2} \ln \frac{M_P}{\mu_0}} = \tilde{a}^2 . \quad (57)$$

where  $\tilde{a}^2 = \lambda^2/s$ . The vacuum energy has the expression

$$\mathcal{E}_0(i) = -2A\lambda^2(\mu_0) = -16\pi^2 A \frac{\tilde{a}^2}{8\pi^2 + 3\tilde{a}^2 \ln \frac{M_P}{\mu_0}} \quad (58)$$

ii)  $\lambda_2(M_P) \ll \lambda_1(M_P)$ . In this case eq.(56) can be approximately integrated, and the solution is

$$\begin{cases} \lambda_1^2(\mu_0) = \frac{\lambda_1^2(M_P)}{1 + \frac{5\lambda_1^2(M_P)}{16\pi^2} \ln \frac{M_P}{\mu_0}} \\ \lambda_2^2(\mu_0) = \frac{\lambda_2^2(M_P)}{\left[1 + \frac{5\lambda_1^2(M_P)}{16\pi^2} \ln \frac{M_P}{\mu_0}\right]^{\frac{1}{5}}} \end{cases} \quad (59)$$

The constraint eq.(38) in the case  $\lambda_2^2(\mu_0) \simeq 0$  will then impose

$$\begin{cases} \lambda_2^2(\mu_0) \simeq 0 \\ \lambda_1^2(\mu_0) \simeq \frac{16\pi^2}{5\ell n \frac{M_P}{\mu_0}} \end{cases} \quad (60)$$

$\lambda_1^2(\mu_0)$  is determined, as in the toy model, by the triviality bound, corresponding to  $\lambda_1^2(M_P) \gg 1$ . We will postpone the question whether the small couplings are exactly zero or not to the next section. There a detailed analysis in the context of the MSSM will show that they indeed are different from zero and a fermion mass hierarchy is generated. This is not relevant for the present analysis as long as the small coupling is negligible compared to the large one, as should be clear from the additive form of the  $\mathcal{E}_0$ , eq.(54).

The vacuum energy is approximately given by

$$\mathcal{E}_0(ii) = -\frac{16\pi^2 A}{5\ell n \frac{M_P}{\mu_0}} \quad (61)$$

Comparing the two energies (58) and (61) we obtain the condition needed in order to have  $\mathcal{E}_0(ii) < \mathcal{E}_0(i)$ . Explicitly we have  $\tilde{a}^2 < 4\pi^2 / \ln \frac{M_P}{\mu_0}$ . Substituting  $\tilde{a}^2$  with the expectation value of the dilaton  $s$ , we get

$$s > \frac{\lambda^2 \ln \frac{M_P}{\mu_0}}{4\pi^2} \quad (62)$$

This is the main difference between a multiplicative-type constraint and an additive type constraint. In the first case, a minimal value for the vev of dilation  $s$  is necessary in order to have the Nambu mechanism, whereas in the second case no condition is needed.

It should be remarked that the RG equations (56) have an infrared fixed point  $\lambda_1^2/\lambda_2^2 = 1$  which corresponds to the symmetric case (i) above. The previous analysis tells us that, if eq.(62) is satisfied, this solution is disfavored

compared with the asymmetric one  $\lambda_2 \ll \lambda_1$  and the infrared fixed point is not reached. A detailed numerical analysis shows that the solution (ii) is the absolute minimum.

Adding a third quark coupling does not change qualitatively the results. The preferred configuration has two very small Yukawas and the third one fixed by the triviality bound. The only change is in eq.(62) where the factor  $4\pi^2$  should be replaced by  $2\pi^2$ . The system will never reach the infrared fixed point  $\lambda_2^2/\lambda_1^2 = \lambda_3^2/\lambda_1^2 = 1$ .

## 4 The case of the Minimal Supersymmetric Standard Model (MSSM).

We now analyse the MSSM model [9] and the possible phenomenological constraints which must be satisfied in order for the Nambu mechanism to work. We first concentrate on the sign of the  $A$  coefficient of eq.(2), in the leading  $\ln \Lambda^2$  approximation and try to be as general as possible regarding the constraint between Yukawas. As long as the constant  $\tilde{a}^2$  in eq.(28) is sufficiently small, eq.(62), the positivity of  $A$  is the signal of a generation of hierarchies, both for additive and multiplicative constraints. We then compute the bottom coupling in the MSSM, neglecting the Yukawas of the first two generations and using the multiplicative constraint eq.(28). The value obtained is naturally small compared with the top mass, although we do not obtain an exponential hierarchy of the Nambu type ( which would anyway be too large to account for the observed ratio of masses ).

The superpotential for the MSSM is given by the formula

$$W = \lambda_U^{ij} Q^i U^{cj} H_2 + \lambda_D^{ij} Q^i D^{cj} H_1 + \lambda_L^{ij} L^i E^{cj} H_1 + \mu H_1 H_2 \quad (63)$$

where  $i, j = 1, 2, 3$  are generation indices and the soft-breaking terms read

$$\begin{aligned} -\mathcal{L}_{soft} = & M^2 (|z_Q|^2 + |z_{U^c}|^2 + |z_{D^c}|^2) + \sum_{k=L, E^c} M_k^2 |z_k|^2 + m_1^2 |z_1|^2 + m_2^2 |z_2|^2 \\ & + m_3^2 (z_1 z_2 + z_1^+ z_2^+) + (\mathcal{A}_U^{ij} z_U^i z_{U^c}^j z_2 + \mathcal{A}_D^{ij} z_D^i z_{D^c}^j z_1 + \mathcal{A}_L^{ij} z_L^i z_{L^c}^j z_1 + h.c.) + \\ & + \frac{M_3}{2} (\lambda_3^A \lambda_3^A + \bar{\lambda}_3^A \bar{\lambda}_3^A) + \frac{M_2}{2} (\lambda_2^i \lambda_2^i + \bar{\lambda}_2^i \bar{\lambda}_2^i) + \frac{M_1}{2} (\lambda_1 \lambda_1 + \bar{\lambda}_1 \bar{\lambda}_1) , \end{aligned} \quad (64)$$

where  $z_x$  is the scalar component of the chiral superfield  $X$ ,  $z_i$  ( $i = 1, 2$ ) are the two Higgs fields and  $\lambda_3^A, \lambda_2^i, \lambda_1$  are the  $SU(3), SU(2), U(1)$  gaugino fields. Keeping only the neutral scalar fields for  $H_1$  and  $H_2, z_1$  and  $z_2$  of vev's  $v_1$  and  $v_2$ , the tree-level scalar potential reads

$$V_0 = (\mu^2 + m_1^2) |z_1|^2 + (\mu^2 + m_2^2) |z_2|^2 + m_3^2 (z_1 z_2 + z_1^+ z_2^+) + \frac{g_1^2 + g_2^2}{8} (|z_1|^2 - |z_2|^2)^2 , \quad (65)$$

where  $g_1$  and  $g_2$  are the  $U(1)$  and  $SU(2)$  coupling constants, respectively. An important parameter of the theory is the angle  $\beta$  defined by  $tg\beta = v_2/v_1$ , expressed after minimization of  $V_0$  in terms of the other parameters by

$$\sin 2\beta = \frac{-2m_3^2}{2\mu^2 + m_1^2 + m_2^2}. \quad (66)$$

As expected, there is no Yukawa dependence of the vacuum energy at tree level, eq.(65) and we must go to the one-loop level. In the leading  $\ln \Lambda^2$  approximation, the vacuum energy is determined by  $StrM^4$  which reads

$$\begin{aligned} \frac{1}{3} StrM^4 &= [4(\mu^2/tg^2\beta + 2M^2) - (g_1^2 + g_2^2)(v_2^2 - v_1^2)] Tr\lambda_U^2 v_2^2 + \\ &+ [4(\mu^2 tg^2\beta + 2M^2) + (g_1^2 + g_2^2)(v_2^2 - v_1^2)] Tr(\lambda_D^2 + \frac{1}{3}\lambda_L^2)v_1^2 + \\ &8\mu Tr(\lambda_U \mathcal{A}_U + \lambda_D \mathcal{A}_D + \frac{1}{3}\lambda_L \mathcal{A}_L)v_1 v_2 \\ &\equiv A_U Tr\lambda_U^2 + A_D Tr(\lambda_D^2 + \frac{1}{3}\lambda_L^2) + 8\mu Tr(\lambda_U \mathcal{A}_U + \lambda_D \mathcal{A}_D + \frac{1}{3}\lambda_L \mathcal{A}_L)v_1 v_2. \end{aligned} \quad (67)$$

The last line defines the parameters  $A_U$  and  $A_D$ . Using the  $Z$  mass expression  $M_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_1^2 + v_2^2)$ , we can rewrite  $A_U$  and  $A_D$  as

$$\begin{aligned} A_U &= 2 [2\mu^2/tg^2\beta + 4M^2 - M_Z^2 + (g_1^2 + g_2^2)v_1^2] v_2^2, \\ A_D &= 2 [2\mu^2 tg^2\beta + 4M^2 - M_Z^2 + (g_1^2 + g_2^2)v_2^2] v_1^2. \end{aligned} \quad (68)$$

In order to decide about the signs of  $A_U$  and  $A_D$  we use the experimental inequality

$$(StrM^2)_{\text{quarks + squarks}} = 4M^2 > M_Z^2. \quad (69)$$

Then  $A_U, A_D > 0$  and the vacuum energy in the leading  $\log \Lambda^2$  approximation has the Nambu form, eq.(2), with  $B = 0$  and an additional linear term which does not change the shape of the vacuum energy as a function of the Yukawas, but will play an essential role in the minimisation process.

The positivity of  $A_U, A_D$  is a direct consequence of supersymmetry and is due to the Yukawa dependent bosonic contributions in (67). In the non-supersymmetric Standard Model the sign is negative and the present considerations would not apply. Using eq.(49) and eq.(50), we obtain the vacuum energy as a function of  $\lambda_U$  and  $\lambda_D$ , which is a paraboloid unbounded from below. If we had no constraint, both Yukawas would tend to the maximally allowed values and no hierarchy would be generated. The role of the constraint, as emphasized in [4] is to restrict the coupling constant parameter space so that the minimum of the vacuum energy is *exactly* where the mass hierarchy occurs.

Because the Nambu factor  $A_D$  for the D-type quarks is three times larger than the corresponding one for the leptons, the heaviest fermion obtained by

minimization is always a quark and not a lepton. In order to decide whether the heavy quark will be of the  $U$  or of the  $D$  type, we must compare  $A_U$  and  $A_D$ . If  $A_U > A_D$ , a  $U$  type quark becomes massive and all the others remain light (in the leading  $\log \Lambda^2$  approximation). Using the definitions in eq.(67), we find

$$A_U - A_D = 2(tg^2\beta - 1)(-2\mu^2 + 4M^2 - M_Z^2)v_1^2. \quad (70)$$

The region in the parameter space where  $A_U > A_D$  is described by the inequality:

$$tg^2\beta > \frac{2M^2 + m_1^2}{2M^2 + m_2^2}. \quad (71)$$

We therefore need a minimal critical value for  $tg\beta$  of order one, which depends on the soft masses, in order to have a heavy top quark. As we will explicitly check, and in a way similar to the toy model considered in the preceding paragraph, the Nambu mechanism is dictated by the sign of  $A_U$  and  $A_D$  and a value of the dilaton vev larger than a critical value. Then eq.(71) states that there is no need of fine tuning in order to understand the hierarchy between the top quark and the other fermions.

In what follows, the Yukawas for the first two generations will be neglected; only the top and the bottom Yukawas, denoted  $\lambda_U$  and  $\lambda_D$  will be considered.

In order to do the low energy minimization of the vacuum energy, we will proceed in two steps. First of all, we will show that a non-trivial minimum for  $\lambda_D$  appears in the MSSM compatible with  $\lambda_D/\lambda_U \ll 1$ . An analytic expression will be derived, as function of the MSSM parameters. A second step, as above, is the comparison of the energy of this extremum with that of the symmetric solution  $\lambda_D = \lambda_U$ .

The RG equations in the case  $\lambda_U(M_P) = \lambda_D(M_P)$  simplify, because in this case  $\lambda_U(\mu_0) = \lambda_D(\mu_0) \equiv \lambda(\mu_0)$  for any  $\mu_0 < M_P$ . It reads ( $g_3$  and  $g_2$  are the  $SU(3) \times SU(2)$  coupling constants)

$$\mu \frac{d}{d\mu} \lambda = \frac{\lambda}{16\pi^2} (7\lambda^2 - \frac{16}{3}g_3^2 - 3g_2^2). \quad (72)$$

Defining [20]

$$\gamma^2(Q) = e^{-\frac{1}{8\pi^2} \int_{M_P}^Q (\frac{16}{3}g_3^2 + 3g_2^2) dt}, \quad (73)$$

the solution of eq.(72) is given by

$$\lambda^2(\mu_0) = \gamma^2(\mu_0) \frac{\lambda^2(M_P)}{1 + \frac{7\lambda^2(M_P)}{8\pi^2\gamma^2(M_P)} \int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q}. \quad (74)$$

In the case  $\lambda_U(M_P) \gg \lambda_D(M_P)$ , the RG equation are approximately given by

$$\begin{aligned}\mu \frac{d}{d\mu} \lambda_U &= \frac{\lambda_U}{16\pi^2} (6\lambda_U^2 - \frac{16}{3}g_3^2 - 3g_2^2) \\ \frac{d}{d\mu} \lambda_D &= \frac{\lambda_D}{16\pi^2} (\lambda_U^2 - \frac{16}{3}g_3^2 - 3g_2^2)\end{aligned}\quad (75)$$

and, for  $\lambda_U(M_P) \gg 1$ , an approximate solution of (75) is

$$\begin{aligned}\lambda_U^2(M_P) &= \frac{1}{\gamma^2(\mu_0)} \frac{\lambda_U^2(\mu_0)}{1 - \frac{6\lambda_U^2(\mu_0)}{8\pi^2\gamma^2(\mu_0)} \int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q} , \\ \lambda_D(M_P) &= \frac{\lambda_D(\mu_0)}{\gamma_3(\mu_0)} e^{(1/16\pi^2) \int_{\mu_0}^{M_P} \lambda_U^2 d\ln Q} .\end{aligned}\quad (76)$$

The constraint (28)

$$\lambda_U(M_P)\lambda_D(M_P) = \tilde{a}^2 \quad (77)$$

in this approximation reads explicitly

$$\frac{1}{\gamma^4(\mu_0)} \frac{\lambda_U^2(\mu_0)\lambda_D^2(\mu_0)}{1 - \frac{6\lambda^2(\mu_0)}{8\pi^2\gamma^2(\mu_0)} \int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q} e^{(1/8\pi^2) \int_{\mu_0}^{M_P} \lambda_U^2 d\ln Q} = \tilde{a}^4 . \quad (78)$$

It is highly non-linear and difficult to deal analytically with . We make the hypothesis that the minimum lies close to the top quasi-infrared fixed point and linearize around this point. A self-consistency check will be performed at the end. First of all, we trade  $\lambda_U$  in favor of a new variable  $\delta$  defined as

$$\delta^2 = 1 - \frac{6\lambda^2(\mu_0)}{8\pi^2\gamma^2(\mu_0)} \int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q , \quad (79)$$

such that

$$\lambda_U^2(\mu_0) = \frac{8\pi^2\gamma^2(\mu_0)}{6 \int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q} (1 - \delta^2) = x_0^2 (1 - \delta^2) . \quad (80)$$

The constraint (78) in the limit  $\delta \ll 1$  gives us

$$\lambda_D^2(\mu_0) = c^2 \delta^2 / x_0^2 , \quad (81)$$

where

$$c^2 = \tilde{a}^4 \gamma^4(\mu_0) e^{-(1/8\pi^2) \int_{\mu_0}^{M_P} x_0^2 d\ln Q} . \quad (82)$$

Thanks to the constraint, we only have one variable,  $\delta$ , to be minimized in the effective potential. The solution  $\lambda_D = 0$  corresponds to  $\delta = 0$ , so we are interested in the small  $\delta$  limit.

The one-loop effective potential, neglecting the logarithmic corrections (we will come back later to discuss their effect) can be cast in the simple form

$$V_1(\mu_0) = V_0(\mu_0) - (A'_U \lambda_U^2 + A'_D \lambda_D^2) - \alpha(\mathcal{A}_U \lambda_U + \mathcal{A}_D \lambda_D) , \quad (83)$$

where we defined the functions

$$\begin{aligned} A'_{U,D} &= (3 \ln \tilde{\mu}_0^2 / 64\pi^2) A_{U,D} \\ \alpha &= (24\mu / 64\pi^2) \ln \tilde{\mu}_0^2 (\lambda_U \mathcal{A}_U + \lambda_D \mathcal{A}_D) v_1 v_2 , \end{aligned} \quad (84)$$

$\tilde{\mu}_0$  being defined as in eq.(53).

Using the explicit expressions for  $\lambda_{U,D}$  (80) and (81),  $V_1$  appears simply as a function at most quadratic in  $\delta$

$$V_1(\mu_0) = cst + (A'_U x_0^2 - \frac{A'_D c^2}{x_0^2} + \frac{\alpha}{2} \mathcal{A}_U x_0) \delta^2 - \alpha \frac{\mathcal{A}_D c}{x_0} \delta . \quad (85)$$

We minimize this expression with respect to  $\delta$ , keeping all the other parameters fixed, particularly the Higgs vev's  $v_1$  and  $v_2$ . Two possibilities arise:

*i)* If

$$\tilde{a}^4 < [x_0^3 (2A'_U x_0 + \alpha \mathcal{A}_U) / 2A'_D \gamma^4(\mu_0)] e^{(1/8\pi^2) \int_{\mu_0}^{M_P} x_0^2 \ln Q} \quad (86)$$

we have a minimum for  $\delta$ , given by

$$\delta = \alpha \mathcal{A}_D c / (2A'_U x_0^3 - \frac{2A'_D c^2}{x_0} + \alpha \mathcal{A}_U x_0^2) . \quad (87)$$

*ii)* If

$$\tilde{a}^4 > [x_0^3 (2A'_U x_0 + \alpha \mathcal{A}_U) / 2A'_D \gamma^4(\mu_0)] e^{(1/8\pi^2) \int_{\mu_0}^{M_P} x_0^2 \ln Q} \quad (88)$$

the extremum becomes a maximum.

Generically (for a large region of the parameter space)  $\delta \ll 1$  if  $\tilde{a}^2 < 1$ . Taking into account that  $\tilde{a}^2$  is related to the gauge coupling constant value at the Planck scale (so to the dilaton vev), this is a reasonable assumption for weakly coupled effective string models. Consequently, from now on we place ourselves in the case *i*).

The ratio of the two Yukawa couplings at this extremum is given by

$$\lambda_D / \lambda_U = \tilde{a}^4 \alpha \mathcal{A}_D \gamma^4(\mu_0) e^{-(1/8\pi^2) \int_{\mu_0}^{M_P} x_0^2 d \ln Q} / (2A'_U x_0^3 - \frac{2A'_D c^2}{x_0} + \alpha \mathcal{A}_U x_0^2) . \quad (89)$$

A parameter space analysis of this relation shows that for a large region and  $\tilde{a}^4 \sim 1/4$  (a phenomenologically reasonable value),  $\lambda_D / \lambda_U \ll 1$  and no large

value for  $\tan\beta$  is needed in order to correctly reproduce the top and bottom masses from the experimental data.

Let us note that the  $\mu$  parameter of MSSM [23] is essential to produce a non-vanishing value for  $\lambda_D$ .

We finally come back to the logarithmic corrections which were neglected in  $V_1(\mu_0)$ . These could be important and change qualitatively the conclusions if they dominate in the small  $\lambda_D$  limit. An explicit computation of the term  $StrM^4\ell nM^2$  shows that the relevant term behaves as  $\lambda_D^4\ell n\lambda_D^2$  and is completely negligible compared to the linear term considered above. So, compared to our original motivation, the toy model of Nambu, the logarithmic corrections play no role in the MSSM and the bottom mass is entirely due to a linear term proportional to the parameter  $\mu$ .

We can now compute the ratio of the two vacuum energies. The simplest case is  $\mathcal{A}_U = \mathcal{A}_D = 0$ , in which case we obtain

$$\left| \frac{\mathcal{E}_0^{\lambda_U=\lambda_D}}{\mathcal{E}_0^{\lambda_U >> \lambda_D}} \right| = \frac{A_U + A_D}{A_U} \frac{6\lambda^2(M_P) \int_{\mu}^{M_P} \gamma^2(Q) d\ln Q}{8\pi^2 \gamma^2(M_P) + 7\lambda^2(M_P) \int_{\mu}^{M_P} \gamma^2(Q) d\ln Q} . \quad (90)$$

Using the fact that  $\frac{A_U + A_D}{A_U} < 2$ , we find that a sufficient condition for the configuration  $\lambda_U >> \lambda_D$  to be energetically preferred is

$$\lambda^2(M_P) < \frac{8\pi^2}{5} \frac{\gamma^2(M_P)}{\int_{\mu_0}^{M_P} \gamma^2(Q) d\ln Q} . \quad (91)$$

In the limit of very small gauge coupling constants, eq.(91) reduces to an inequality of the type (62) for the dilaton. In the general case with  $\mathcal{A}_U, \mathcal{A}_D$  different from zero, the equivalent of the equation (91) becomes more involved, but we always have an upper bound for  $\lambda^2(M_P)$  which is equivalent to a lower bound for the dilaton  $s$ .

## 5 Conclusions

The aim of this paper is a dynamical understanding of the hierarchy between the mass of the top quark and the other fermions. The central hypothesis is that at the tree level of supergravity the theory has flat directions which are lifted by the breaking of supersymmetry. If the mass of the corresponding moduli is very small, of the order of the electroweak scale, and if the low-energy Yukawa couplings are non-trivial homogeneous functions of the moduli, the Yukawas can be regarded as dynamical variables at low-energy. The homogeneity property is natural in the context of the effective string models, which guarantee also the existence of the flat directions. Our ignorance about the supersymmetry breaking mechanism is hidden in the soft-breaking parameters, which in turn will fix the couplings by the minimization process. The procedure can be viewed

also as a way to compute ratios of the expectation values of the moduli fields ignoring the precise mechanism of supersymmetry breaking.

The existence of constraints between the low-energy couplings is automatic *if* the number of couplings which are *non-trivial* functions of the moduli is greater or equal to the number of moduli. The most interesting case is when the model has the same number of couplings and moduli. In this case only one constraint is obtained at the Planck scale, to be evolved down to the scale  $\mu_0 \sim M_{susy}$ .

Assuming Kähler type transformations for the Kähler potential under the duality symmetries imposes a multiplicative structure to the constraints at the Planck scale. To obtain a Veltman-like additive constraint, we must give up the geometrical structure of the Kähler potential, keeping only the axionic symmetries and a diagonal scale invariance for the moduli. The multiplicative constraints generically put lower limits on the dilaton in order for the mechanism to work, which in superstring-inspired supergravity, allow us to consider only the perturbative regime of the string.

When we apply this to the MSSM, we find that, due to supersymmetry, the functional dependence of the vacuum energy is in a first approximation as in the toy model of Nambu, eq.(2) with  $A > 0$ ,  $B = 0$ . An additional linear term plays an important role in the minimization process. The mechanism predicts a heavy top quark if  $tg\beta$  has a lower limit of the order of one, given in eq.(71), and a heavy bottom quark if this limit is violated. The heavy fermion can never be a lepton due to the small coefficient in front of its Yukawa coupling in the vacuum energy. A lower limit on the dilaton vev must be imposed; its explicit value was obtained in the simplifying case of small gauge coupling constants and vanishing trilinear soft-breaking terms. The condition becomes more involved in the general case.

We computed analytically the bottom Yukawa coupling neglecting the Yukawas for the first two generations, using the multiplicative constraint (77) at the Planck scale. The effective potential at a low scale  $\mu_0 \sim M_Z$  is minimized with respect to  $\lambda_U$  and  $\lambda_D$ , taking into account the constraint translated at the scale  $\mu_0$  with the help of the RG equations. The top mass is very close to the infrared effective fixed-point value, which is due essentially to the fact that the minimization forces  $\lambda_U(M_P) \gg 1$ . The presence in the effective potential of a term linear in the Yukawas and proportional to the  $\mu$  parameter of MSSM turns out to be essential for the stabilization of the bottom Yukawa to a small, non-vanishing value.

The bottom mass is naturally small but not exponentially suppressed as in the Nambu example. The computed value is compatible with the existing data for a large allowed region of the parameter space of the MSSM.

We insist on the fact that  $tg\beta$  can be of order one and still the ratio  $m_b/m_t$  can easily be made small, due to the small value of the corresponding Yukawas.

A complete phenomenological analysis would, of course, require the inclusion of all the Yukawas. The difficult part of this program is probably in extracting

the correct constraint(s) from the underlying string level. The large number of soft-breaking terms can be substantially reduced by imposing some universal boundary conditions at the Planck scale. In this way, the mechanism acquires a predictive power and can be confronted with the known phenomenology, all the Yukawas being dynamically determined.

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